ADDITIONAL MATHEMATICS

Paper 4037/11
Paper 11

Key Messages

Candidates should be reminded to read each question carefully. In particular, they should pay attention to the required form of an answer. They should be aware that an exact answer is one written as a surd and / or in terms of π and that decimal answers should not be given in these cases. Candidates should also check that they have completed all that is required for the question and taken note of restrictions on the answer such as 'positive'.

General Comments

The paper resulted in a good range of responses showing that many candidates had worked hard and understood the syllabus objectives, being able to apply them appropriately. Candidates appeared to have no timing issues. Candidates made good use of the space provided on the paper and responses were generally straightforward to mark with a good standard of presentation. Candidates would benefit from more practice in questions involving vectors.

Comments on Specific Questions

Question 1

Candidates should be aware that for both parts of this question an answer in surd form was required.

(i) Of the two approaches used, equating the discriminant to zero and solving the resulting quadratic equation was the more successful. Candidates who differentiated to find the gradient did not always equate it to *k*. Candidates should be aware that a positive value of *k* was required.

(ii) Candidates who had not obtained an answer in surd form for the first part could not make progress in this second part. Candidates who had a correct surd from the first part usually went on to rationalise the negative reciprocal.

Answers: **(i)** $4+2\sqrt{5}$ **(ii)** $1-\dfrac{1}{2\sqrt{5}}$

Question 2

Many candidates were fully successful in this question. Most obtained a correct equation using $x = 3$ but not all candidates understood that differentiation was required before substitution of $x = 1$. Candidates who obtained two correct equations nearly always went on to solve them correctly.

Answer: $a = 12$, $b = -27$

Question 3

(a) This question was generally well done with most candidates understanding the rules of indices.

(b) (i) To make progress with this question, candidates had to appreciate that $(t-2)^{\frac{3}{2}}$ is equal to $(t-2)(t-2)^{\frac{1}{2}}$ and thus identify a common factor. Many candidates made an incorrect assumption at this initial stage and could not obtain a linear factor. Candidates who made a good start sometimes lost marks through carelessness in simplifying $4 + 5(t - 2)$.

(ii) Candidates with a correct form of answer in **part (i)** went on to use it to solve this part with the occasional omission of $t = 2$. However, having been unsuccessful in the previous part, many candidates did not attempt this question.

Answers: **(a)** x^3y^7 **(b)(i)** $(t-2)^{\frac{1}{2}}(5t-6)$ **(ii)** 2 and $\frac{6}{5}$

Question 4

(a) (i) Candidates should be aware that the exponential function is strictly greater than zero for all real values. A significant number of candidates mistakenly believed that either $x = 0$ would lead to a minimum value of e^{-4x} or that the range was all real numbers.

(ii) Most candidates knew how to approach this question and either switched x and y at the beginning or made x the subject and switched x and y finally. There was some carelessness by the candidates who took the latter approach, with loss of the negative sign or other transcription errors. Few candidates appreciated that for $\ln\frac{x-5}{3}$ to exist, x had to be greater than 5. Candidates who had obtained $y > 5$ as the range of f in **part (i)** often went on to correctly identify $x > 5$ as the domain of the inverse function.

(b) Most candidates identified the correct order of operations and many of them went on to deal with the natural logarithm correctly to obtain a correct solution. A small number of candidates did not select the positive solution.

Answers: **(a)(i)** $f(x) > 5$ **(ii)** $f^{-1}(x) - \frac{1}{4}\ln\frac{x-5}{3}$, $x > 5$ **(b)** 1.55

Question 5

Questions involving vectors are a weakness for a significant number of candidates.

(a) (i) Although there were many good solutions, a significant number of candidates misunderstood how to obtain $\overrightarrow{OM}$ and made errors such as using $\overrightarrow{MA}$ instead of $\overrightarrow{AM}$.

(ii) A good number of candidates used $\frac{5}{2}$ or $\frac{2}{5}$ or dealt correctly with the ratios in an equivalent method but some candidates confused $\overrightarrow{OB}$ with $\overrightarrow{MB}$ and used $\frac{3}{2}$ or $\frac{2}{3}$.

(b) Candidates should be aware of correct vector notation. The use of **i** and **j** within a column vector was sometimes seen in this question.

(i) Many candidates did not appreciate that 'has the same direction' implied that **p** was a multiple of $-10\mathbf{i} + 24\mathbf{j}$. The use of the word magnitude in the question led to many calculations of $|-10\mathbf{i} + 24\mathbf{j}|$ but this was not always used with 39 to produce a factor.

(ii) Candidates found this part difficult and tended to concentrate on the 'magnitude of 12' rather than 'parallel to the positive y-axis' not realising that an **i** component of zero was the key to proceeding towards an answer.

(iii) Most candidates knew how to find the magnitude of their answer to the previous part. As candidates had been asked to show a given answer, an intermediate step was required between $\sqrt{4500}$ and $30\sqrt{5}$.

Answers: **(a)(i)** $\frac{1}{2}(\mathbf{a}+\mathbf{c})$ **(ii)** $\frac{5}{4}(\mathbf{a}+\mathbf{c})$ **(b)(i)** –15**i** + 36**j** **(ii)** 30**i** – 60**j** **(iii)** $k = 30$

Question 6

Candidates should be reminded that it is not appropriate to use angles in degrees in circular measure questions. Answers obtained using degrees are likely to lose accuracy because of the extra steps involved.

(i) The majority of candidates knew how to connect the area of the sector to the angle in radians. However, not all appreciated that an answer to 3 decimal places was required in order to show angle *AOB* rounded to 2 decimal places.

(ii) There were many good solutions with candidates showing a good understanding of the subject matter and of the approach required for this particular problem. However, candidates would benefit from practice in selecting the most efficient methods, as methods involving several steps invariably led to accuracy errors. For example, in this question it would be better to use $\frac{1}{2}ab\sin C$ for the area of the triangle rather than attempting $\frac{1}{2}$ base x height. Many candidates used the cosine rule to find *AB* but more care was needed in evaluation in some responses. Candidates using the half angle to find *AB* sometimes lost marks through premature rounding.

(iii) A majority of candidates were able to find the length of the arc but not all candidates formed a correct plan to find the required perimeter.

Answers: **(ii)** 78.4 or 78.5 **(iii)** 61.7

Question 7

Most candidates knew they had to differentiate, but differentiation of the composite function posed difficulties for some. Most candidates knew to equate to zero but the solution of $x = 0$ was sometimes overlooked. Most candidates chose to use the second derivative method to identify the nature of the stationary point. Many did not realise that the product rule was required to do this. Those that did use the product rule had difficulties coping with the differentiation of $(3x^2+8)^{\frac{2}{3}}$, so few candidates reached a correct conclusion from a correct derivative. Candidates using a method involving the gradient on either side were required to show adequate evidence.

Answer: Minimum at (0, 32)

Question 8

(i) Candidates were asked to use the axes provided and therefore expected to use values of *x* from –1 to 6, but many candidates only used positive *x* values. Candidates showed a good knowledge of the shape of the graph of the modulus of a linear function but a more careful approach was sometimes required. Ruled lines should be used and these should meet as a V on the *x*-axis. Candidates using a plotting approach were less likely to appreciate this. Many attempts for the quadratic curve started at (0, 0). Candidates would benefit from increasing their knowledge of how graphs of functions should look rather than relying on plotting.

(ii) There were many good solutions but not all candidates knew how to solve an equation of that form. Most candidates obtained $\frac{9}{2}$ from $2x – 5 = 4$ but some did not look for a second solution. Others required more care with signs when solving $–(2x – 5) = 4$ and solving $2x – 5 = –4$.

(iii) Candidates who used a verification method often only substituted one value of *x* in the quadratic expression to obtain *y* = 4. Candidates who formed an equation using $4 \times 9 = 80x - 16x^2$ did not always provide a convincing solution. Those who solved $\pm 9(2x-5) = 80x - 16x^2$ to find *x* values often forgot that they had to show that *y* was equal to 4.

(iv) Candidates rarely associated the solution for this part with the answers obtained in previous parts. In particular, most candidates had sketched a graph that could help them see the answer to this part. Those who started afresh and found solutions for $\pm 9(2x-5) = 80x - 16x^2$ had difficulty obtaining a correct range for *x*, as extra solutions had been found.

Answers: **(ii)** $\frac{9}{2}$, $\frac{1}{2}$ **(iv)** $\frac{1}{2} \leqslant x \leqslant \frac{9}{2}$

Question 9

(i) Candidates should be aware that full working has to be shown when they are asked to show a result. A substitution of $\sec^2 \frac{x}{3} - 1$ for $\tan^2 \frac{x}{3}$ had to be seen, rather than implied, in this case.

(ii) This was usually well answered but some candidates gave $\frac{1}{3}\tan\frac{x}{3}$.

(iii) A good number of candidates used a multiple of $\tan\frac{x}{3}$, clearly attempting to use the previous parts. Credit was given for steps in terms of $\tan\frac{\pi}{3}$ and $\tan\frac{\pi}{6}$ leading to an exact answer but not for work in decimals. Candidates should be aware that in this case the exact answer was $8\sqrt{3} + \frac{\pi}{2}$ and that a decimal answer was not required.

Answers: **(ii)** $3\tan\frac{\pi}{3} + c$ **(iii)** $8\sqrt{3} + \frac{\pi}{2}$

Question 10

Candidates should be aware that the results for differentiation of standard functions are not provided in the formula list and have to be learned beforehand.

(a) Most candidates preferred to use the quotient rule for this question and many good responses were seen with a correct form of the rule being used in the majority of responses. Some candidates lost marks by using an incorrect derivative for e^{3x}.

(b) (i) Although there were many good responses, not all candidates recalled the derivatives of the trigonometric functions. Some responses tried to treat each part as a composite function or the whole expression as a product. A significant number of candidates lost the accuracy mark by having their calculator set for degrees rather than radians. Candidates should be aware that standard derivatives of trigonometric functions apply for angles measured in radians.

(ii) Many candidates successfully related the rates of change using their answer to **part (i)** but not all candidates attempted this part.

Answers: **(a)** $\frac{dy}{dx} = \frac{3e^{3x}(4x^2+1) - 8xe^{3x}}{(4x^2+1)^2}$ **(b)(i)** –5 **(ii)** –2

ADDITIONAL MATHEMATICS

Paper 4037/12
Paper 12

Key messages

Candidates should be reminded to read each question carefully. In particular, they should pay attention to the required form of an answer. They should be aware that an exact answer is one that is written either as a surd and/or in terms of π or a fraction and that decimal answers should not be given in these cases. Candidates should also check that they have completed all that is required for the question and taken note of restrictions on the answer such as 'positive'. Candidates should also realise that the word 'Hence' implies that work done in the previous part of the question is to be made use of. The mark allocation for each question part gives a good measure of the amount of work to be done.

General comments

Some excellent scripts were seen. In general there was a good range of marks from the cohort with it being evident that many candidates were well prepared for the examination. Candidates appeared to have had enough time to complete the paper. Some candidates did not have enough room for their work in the space provided. Candidates should be encouraged to think about the structure of their response in the limited space that is available and try to set out their response in a neat and orderly fashion. Rather than write in margins or continue into the spaces for the following questions, candidates should either make use of any blank pages in the examination booklet or request an extra sheet or two of paper. A statement to the effect that the question is continued elsewhere will ensure that the work is located easily and given any credit due.

Comments on specific questions

Question 1

A straightforward question on shading given regions on Venn diagrams meant that many candidates were able to make a good start to the paper. There were many completely correct responses showing that candidates have a good understanding of the syllabus requirements.

Question 2

Most candidates realised that they needed to differentiate the given equation as a quotient. This question is an example of where candidates should take note of the word 'exact'. Too many candidates resorted to a calculator and gave a decimal answer rather than show all their working with the substitution $x = 3$ and subsequent simplification. Many candidates did attempt this simplification but made simple errors which resulted in an incorrect answer. Some attempts at differentiation of a product were made, usually with less successful results. The form of the question was intended to guide candidates to the use of a quotient rather than a product when differentiating.

Answer: $\frac{11}{112}$

Question 3

Many candidates still have problems when dealing with basic vectors and how a vector can be formed.

(a) Although many correct responses were seen, it was evident that many candidates were unaware of, or did not make use of, the fact that a vector is equal to the product of its magnitude and a unit direction vector. Finding the unit direction vector appeared to be the main problem in this part of the question.

(b) To obtain the basic vector in this part of the question, candidates were required to make use of basic trigonometry to find the magnitude of the vector in each of two perpendicular directions. A simple sketch would have been of benefit to many candidates to enable them to visualise what was required.

Answers: **(a)** $3\mathbf{i}-6\mathbf{j}$ **(b)** $\sqrt{3}\mathbf{i}+\mathbf{j}$

Question 4

Many completely correct responses to this question were seen. There were the occasional slips with signs and the occasional slip resulting from poor arithmetic when simplifying the terms. It is evident that candidates are more aware of both the negative and fractional parts of terms in binomial expansions.

Answer: $n=4$, $a=-18$ and $b=\frac{3}{2}$

Question 5

Some candidates appeared to find the concept of kinematics difficult. Being able to distinguish between speed and velocity, and distance and displacement is essential in order to gain maximum marks. Also looking at the mark allocation for each part of a question of this type should give candidates a good idea of the amount of work required.

(i) Candidates were simply required to differentiate the displacement equation with respect to time. The question was testing not just knowledge of kinematics but the ability to apply calculus appropriately to trigonometric functions.

(ii) The mark allocation together with the use of the word 'Hence' were meant to help indicate to candidates that this part of the question was one which could be done with the minimal amount of work. There were too many candidates who either did not relate the word maximum to the trigonometric function they had obtained in **part (i)** without using further calculus or take heed of the request for the speed of the particle.

(iii) Most candidates realised that they need to differentiate again and equate their result to zero in order to obtain an expression for the acceleration of the particle. It was evident that many candidates do not realise that when dealing with trigonometric functions and calculus that any angles are in radians unless otherwise stated. There were too many answers of 30 rather than the required $\frac{\pi}{6}$.

(iv) Correct answers were in the minority with many candidates not realising that a distance rather than a displacement was required. Again, the mark allocation was meant to indicate to candidates that not much extra work needed to be done.

Answers: **(i)** $-12\sin 3t$ **(ii)** 12 **(iii)** $\frac{\pi}{6}$ **(iv)** 4

Question 6

(i) Many correct proofs were seen, with most candidates opting to express each term in terms of either $\sin\theta$ or $\cos\theta$. Other methods making use of either $\sec^2\theta = 1+\tan^2\theta$ or $\mathrm{cosec}^2\theta = 1+\cot^2\theta$ were equally acceptable and successful. As candidates are required to show the given result, it is important that they show each step of their working in order to gain full marks. Equally important is the need to write out the trig terms correctly. There were some candidates who chose to work through the entire proof omitting θ, which may have saved them a small amount of time but lost them marks as what was written down was meaningless.

(ii) Whilst many candidates attempted to make use of the result to **part (i)**, many did not introduce a double angle. Of those that did, many correct responses were seen.

Answer: **(ii)** $\frac{\pi}{6}$

Question 7

(i) It is important that candidates read the question carefully. In this question many candidates gave an equation of a straight line and did not give the values of *A* and *b* as required. Some candidates did not notice that base 10 logarithms rather than base e logarithms were involved. Many candidates did however obtain the correct value for *b*, usually by finding the gradient of the straight line. Problems occurred when trying to find the value of *A*. Many candidates did not really appreciate the form of the straight line and wrote down and used incorrect equations, for example $\lg 2.2 = \lg A + 0.5b$ rather than the correct $2.2 = \lg A + 0.5b$.

(ii) Even though many candidates had incorrect values for *b* and/or for *A,* most were able to apply their values correctly in either $y = A\left(10^{bx}\right)$ or $\lg y = \lg A + bx$ to obtain a method mark.

(iii) Even though many candidates had incorrect values for *b* and/or for *A,* most were able to apply their values correctly in either $y = A\left(10^{bx}\right)$ or $\lg y = \lg A + bx$ to obtain a method mark. However, it should be noted that answers to three significant figures are required and in this part of the question some candidates lost the final mark with answers of 0.69 and in some cases 0.7.

Answers: **(i)** $b = 3$, $A = 5.01$ **(ii)** 315 or 316 **(iii)** 0.693

Question 8

Many candidates were able to obtain a good score for this question provided they realised that **part (a)** concerned permutations and **part (b)** concerned combinations.

(a) **Parts (i), (ii)** and **(iii)** were answers that were simply written down with no working expected. The majority of these answers were correct with candidates realising that some form of permutations were needed. In **part (iv)** the mark allocation should have alerted candidates to the need to consider more than one step/case in their response. There were different approaches, many of which were successful, but often candidates did not consider each of the steps that were needed for their chosen method. Candidates are to be encouraged to write down what they are attempting to do which will make it easier for an examiner to award method marks rather than trying to decipher what has been intended from lots of sometimes apparently random numbers. Credit was given for partial solutions.

(b) As in **part (a)**, **parts (i)** and **(ii)** were either correct or incorrect, with little work, if any, expected as a response. Again, in **part (iii)**, the mark allocation was meant to guide candidates to the use of a method involving more than one step. Again it was helpful if candidates wrote down what they were intending to find. In this part, it was expected that candidates consider the number of ways when the brother and sister were included together with the number of ways when the brother and sister were not included. Many candidates only considered the number of ways when the brother and sister were included, but were still given credit for this partial answer.

Answers: **(a)(i)** 2520 **(ii)** 360 **(iii)** 1080 **(iv)** 420 **(b)(i)** 4800700 **(ii)** 26460 **(iii)** 278806

Question 9

(a) (i) This part of the question was intended to test the candidates' knowledge of the vocabulary that is used when dealing with matrices. Most candidates gave a correct response.

(ii) Most candidates were able to do the appropriate matrix multiplication and obtain a matrix of the correct order together with the correct elements. There were the occasional arithmetic slips.

(b) (i) Very few incorrect inverse matrices were seen.

(ii) Candidates were instructed to use their inverse matrix. Those candidates that ignored this instruction and just solved a pair of simultaneous equations did not gain any credit as this question was intended to test knowledge of solution of simultaneous equations using a matrix method. It was important that candidates used the correct rearrangement of the two equations together with the correct order of multiplication by the inverse matrix (in this case pre-multiplication). Many completely correct solutions were seen.

Answers: **(a)(i)** 3×2 **(ii)** $\begin{pmatrix} 6 & -6 \\ 5 & 2 \\ 19 & -8 \end{pmatrix}$ **(b)(i)** $\frac{1}{13}\begin{pmatrix} -7 & 12 \\ -4 & 5 \end{pmatrix}$ **(ii)** $(34, 12)$

Question 10

(i) There was a mark allocation of one, which should have informed candidates that very little work was required. It is expected that candidates make use of radians throughout a question of this type. Making use of degrees and then changing to radians results in unnecessary extra work and often inaccurate answers. The use of radians is a syllabus requirement. Too many candidates attempted to use degrees and were often unsuccessful in gaining the correct answer. A simple use of the arc length was expected, an answer that could be written down with little or no working. Too many candidates did not read the question carefully and chose to find angle *AOB* rather than angle *AOD*.

(ii) Many candidates made use of a correct method and found the angle *DOC*, gaining 2 marks. Fewer candidates gained full marks for this part of the question as they did not work to more than two decimal places or show that they had worked to more than two decimal places. The question requirement was to show that the angle was 1.43 correct to two decimal places, not just 1.43 using figures that were already rounded to two decimal places.

(iii) Many correct solutions were seen, with candidates making use of the given answer to **part (ii)**.

(iv) This part was found more difficult. Many candidates did not appear to have a logical approach to their solution or, if they did, it was not always clear. There were many different ways a correct answer could be obtained. Candidates are to be encouraged to write down exactly what they are doing. In this case it would have been useful if some candidates had stated which figure their areas referred to. Too many solutions consisted of masses of figures with no apparent method or no apparent shape, sector, rectangle or segment ever being mentioned specifically. However there were candidates who stated exactly what they were doing, making it easier to award the appropriate marks.

Answers: **(i)** 0.5 **(iii)** 33.5 **(iv)** 42.8

Question 11

(i) It was intended that candidates integrate the given derivative with respect to *x* and then make use of the given point to determine the value of the arbitrary constant. There were a pleasing number of correct solutions. Some candidates forgot to make use of the given point as they had not introduced an arbitrary constant. There were a small minority of candidates who tried to find the equation of a straight line using the value of the derivative at the point given. This was clearly a complete misunderstanding of what was required. There were also examples of incorrect integration of the exponential function which often resulted in candidates gaining few, if any marks.

(ii) Many candidates were able to gain the first three marks by obtaining a result of $\frac{1}{2}+\frac{1}{2}\ln 2$ using a correct second derivative and then omitting to finish their work by obtaining the required form. This again highlights the importance of reading the question carefully and ensuring that it has been answered completely. Some candidates who had not done well in **part (i)** did not attempt this part. Others were unable to differentiate the exponential function correctly.

Answers: **(i)** $\mathrm{f}(x)=\frac{1}{2}e^{2x-1}+3$ **(ii)** $\frac{1}{2}+\ln\sqrt{2}$

ADDITIONAL MATHEMATICS

Paper 4037/21
Paper 21

Key messages

In order to do well in this paper, candidates need to show full and clear methods in order that marks can be awarded. In questions where the answer is given, candidates are required to show that it is correct and fully explained solutions with all method steps shown are needed. In such questions candidates are encouraged to use consistent notation such as using the same variable throughout a solution. In questions that state that a calculator should not be used omitting method steps often results in full credit not being given for a solution. Combining the steps directly into a simplified form, such as that produced by a calculator, cannot be credited. In questions that require a solution of several steps, clearly structured and logical solutions are more likely to gain credit. Candidates should be encouraged to write down any general formula they are using as this reduces errors and is likely to improve the accuracy of their solutions. In particular, method marks cannot be given for solving an incorrect equation when the solutions are taken directly from a calculator, without showing any working. When a diagram or graph is required then they should be completed in full and as accurately as possible. A ruler should be used to draw straight line graphs.

General comments

Some candidates produced high quality work displaying wide-ranging mathematical skills, with well-presented, clearly organised answers. This meant that solutions were generally clear to follow. Other candidates produced solutions with a lot of unlinked working, often resulting in little or no credit being given. More credit was likely to be given when a clear sequence of steps was evident. Quoting a formula which referred to only part of the previous line then applying it on the next line led to candidates confusing themselves. This was particularly evident on **Question 11(i)**.

Questions which required the knowledge of standard methods were done well. Candidates had the opportunity to demonstrate their ability with these methods in many questions. Most candidates showed some knowledge and application of technique. The majority of candidates attempted most questions, demonstrating a full range of abilities.

Some candidates need to improve their reading of questions and keep their working relevant in order to improve. Candidates should also read the question carefully to ensure that, when a question requests the answer in a particular form, they give the answer in that form. Candidates should ensure also that each part of a question is answered and the answer clearly identified. When a question demands that a specific method is used, candidates must realise that little or no credit will be given for the use of a different method. They should also be aware of the need to use the appropriate form of angle measure within a question. When a question indicates that a calculator should not be used, candidates must realise that clear and complete method steps should be shown and that the sight of values clearly found from a calculator will result in the loss of marks.

Where an answer was given and a proof was required, candidates needed to fully explain their reasoning. Omitting method steps in such questions resulted in a loss of marks. Working from both sides and so treating an identity like an equation is not a valid way to prove a given result. Candidates should work from the left hand side to arrive at the result stated on the right hand side.

Candidates should take care with the accuracy of their answers. Centres are advised to remind candidates of the rubric printed on the front page of the examination paper, which clearly states the requirements for this paper. Candidates need to ensure that their working values are of a greater accuracy than is required in their final answer.

Candidates should be advised that any work they wish to delete should be crossed through with a single line so that it can still be read. There are occasions when such work may be marked and it can only be marked when it is readable.

Comments on specific questions

Question 1

Candidates were required to integrate the given equation then use the coordinate point to find the constant of integration and the correct equation for the curve. This was carried out accurately by the majority of candidates. Some candidates found the tangent to the curve through the given point. This was also often seen following or alongside a correct integration. Others omitted "+ *c*" from their integral or made an arithmetic error following substitution.

Answer: $y = x^4 + x - 1$

Question 2

(a) As the question stated that a calculator must not be used, every step in the working needed to be shown clearly, even if it seemed obvious to the candidate, to demonstrate that the instruction had been followed. This applied both to combining or simplifying surd terms and to mental long multiplication. The best answers broke down all the surds into component factors, preferably prime factors, before simplifying and combining the parts. Candidates were usually successful in arriving at the correct answer, but omission of steps in finding $18\sqrt{2}$ frequently saw the deduction of up to two marks.

(b) Candidates were generally able to arrive at an expression for *x*. On occasion, arithmetic errors were seen in this process. Rationalisation of the denominator was usually correctly attempted. As in **part (a)**, sufficient evidence was required to show that the rationalisation was not completed by a calculator, with explicit multiplication, particularly of the numerator, required. Some candidates chose to square both sides of the initial equation and use the quadratic formula. This method rarely resulted in full marks as candidates did not break down the square root sufficiently to give evidence of not having used a calculator. Those successful in this often omitted to reject one of the solutions.

Answers: **(a)** $27\sqrt{2}$ **(b)** $15 + 8\sqrt{3}$

Question 3

(i) Knowledge of the derivative of a function of ln was evident for many candidates. There were slips by some of these in finding the numerator. Several candidates attempted manipulation of logs or the use of the product rule. Neither of these was appropriate to this question.

(ii) Having found a derivative in **part (i)** it was necessary to substitute in *x* = 3 and multiply the answer by *h* thus applying the rule for approximation for small increments. Many candidates appeared unaware of this and substituted 3 and 3 + *h* into the original expression. There were also instances where 3 + *h* was substituted into the derivative.

Answers: **(i)** $\dfrac{2x}{x^2+1}$ **(ii)** $\dfrac{6}{10}h$

Question 4

(a) (i) There were many correct answers to this part. Although there was a clear statement in the question that the angle was measured in degrees, many gave an angle of the correct magnitude in radians.

(ii) This part was carried out more successfully with the vast majority stating the correct answer. Where candidates attempted to show a calculation instead this usually led to an answer of 4 from $7 - 3$ or 10 from 7 + 3.

(b) Many candidates found this part straightforward and just wrote down the correct answer with no working required. It should be noted that as a minimum $g(x) = \sin x$ needed to be stated for full marks to be achieved. Those candidates who attempted intermediate working were often the least successful as their manipulation often started from an assumption that the graph was a transformation of the curve stated in **part (a)**.

Answers: **(a)(i)** 36 **(ii)** 7 **(b)** $y = 5\sin 4x + 7$

Question 5

(i) This question required an application of the binomial expansion and candidates were often successful in gaining full marks. There were occasions where the powers of *a* were omitted or the expansion was incomplete with brackets remaining around $(ax)^n$ in each term. The binomial coefficients were usually correct. The instruction to give the terms in ascending order was almost always followed.

(ii) If the previous part had been correctly attempted then this part was almost always correct. The most popular method was to equate coefficients and solve the resulting equation, occasionally with an additional solution of $x = 0$ found. Equating terms rather than coefficients left candidates with an x in their working. It was not sufficient at this stage to either try to cancel this or to ignore it altogether.

(iii) This was an instance where reading the question carefully was essential. Candidates were asked to use their expansion and not simply insert 1.97 into the original expression. Good answers went back to the start of the question and equated $(2 + ax)$ with 1.97 as was expected. Some evidence of the candidate having attempted the substitution was also required as the value 15.1 had been given. Values for individual terms or a summation to 15.06… was sufficient for this.

Answer: **(i)** $16 + 32ax + 24a^2x^2 + 8a^3x^3 + a^4x^4$

Question 6

(i) This part was usually answered correctly with a matrix of the correct order and only occasional slips in the elements. Errors in this part, such as giving a matrix of the incorrect order inevitably led to marks being lost later in the question.

(ii) If the previous part was correct then it usually followed that this part was as well. There were only rare slips in the matrix multiplication.

(iii) Many candidates had some idea of what **LM** represented but could not give an answer that was precise enough. Some reference needed to be made to the types of ticket being sold as well as to totalling these across the cinemas.

(iv) As stated above, if **M** was not of the correct order, it was highly unlikely that **N** would be as candidates tried to write it so that a multiplication was possible. This multiplication was not required, only what it represented, although the nature of the answer, being a single element, did allow many to draw a better conclusion than in **part (iii)**.

Answers: **(i)** $(\mathbf{M} =)\begin{pmatrix} 90 & 10 & 30 \\ 0 & 45 & 0 \\ 25 & 0 & 15 \\ 10 & 0 & 100 \end{pmatrix}$ **(ii)** $(125\ 55\ 145)$ **(iii)** The total numbers of each type of ticket sold by all four cinemas **(iv)** $(\mathbf{N} =)\begin{pmatrix} 5 \\ 4 \\ 3 \end{pmatrix}$ The total income of all four cinemas

Question 7

(a) This proved to be one of the best answered parts of the paper with many candidates correctly shading both diagrams. In most cases there was little ambiguity shown as to which areas candidates were indicating.

(b) (i) This part was well answered by the majority of candidates, with a fairly even split between those who stated the set was empty and those who said the number of elements was zero. Adding additional brackets was not uncommon but usually led to a set containing the empty set. There were a few candidates who mixed the notations using the empty set in place of a zero. Only a few candidates used the union, rather than the intersection, notation.

(ii) This question required that the Venn diagram be completed and that the appropriate number of students be found. It was therefore necessary both to include the value 17 on the diagram and to identify it as the required answer. A significant number of candidates did one or other but not both. Nearly all candidates were able to produce a diagram with the appropriate intersections or showed blanks or zeros correctly on a standard three set diagram. An algebraic approach to the solution was not common and when used often led to an incomplete diagram or candidates who did not realise that certain sections were zero.

Answers: **(b)(i)** $\text{n}(M \cap D) = 0$ or $M \cap D = \varnothing$ **(ii)** 17

Question 8

This question tended to be either very well attempted with all or most parts correctly calculated with the minimum of working required or candidates showed insufficient understanding to identify the appropriate possible ways the desired outcome could be achieved. There was also confusion as to when to use combinations or permutations. A factor of two was often omitted from the calculation of one or more parts.

(a) The correct answer of 870 was often calculated correctly and in some cases simply stated. 435 was almost as common an answer.

(b) (i) This could be solved by multiplying the number of ways in which a mathematics teacher could be chosen by the number of ways in which the other ten players could be chosen from the fourteen remaining. This was the most popular approach and was frequently correct except when the choice of mathematics teacher was assumed to be 1. There was a longer method involving breaking down the five possible cases and adding their combinations. While this was lengthy by comparison it was rarely incorrect and did have the advantage of having calculated the two cases required in the next part**.**

(ii) It was rare for this not to be correct if **part (i)** had been correct initially. It was also usual for solutions to gain either full marks or no marks at all. Many candidates identified the two possible cases and added their combination correctly. Some candidates chose the computer teachers from the fourteen remaining rather than the nine computer teachers. A complementary method was rarely used.

Answers: **(a)** 870 **(b)(i)** 2002 **(ii)** 1092

Question 9

(i) The vast majority of candidates were able to make a successful start to this question by eliminating one variable, usually *y*. Errors in expansion of the brackets or in collecting terms meant that the correct quadratic was not always found. It was still possible to gain credit for the solution of the candidate's quadratic as long as the candidate showed the method of their solution. Where factorisation was attempted or the quadratic formula was shown this was easily checked. When a candidate relies on their calculator and therefore shows no working their method cannot be checked and no further credit can be awarded.

(ii) As the equation of *AB* was given, the gradient of the line and hence the perpendicular line could be stated. Many candidates chose to calculate the gradient from their *A* and *B* which worked well if these were correct. As a consequence of errors in **part (i)** this frequently led to an incorrect gradient. Candidates could still find an appropriate equation and gain credit if they found the mid-point of their *AB*. The bisector part of the question was often misunderstood or misread and a perpendicular through *A* or *B* was given. The question asked for a particular form for the equation and while the format was often correct the use of integers only was not always evident.

Answers: **(i)** $\left(-\frac{1}{3}, \frac{8}{3}\right)$ $(1, 0)$ **(ii)** $6y - 3x = 7$

Question 10

(i) Most candidates were able to plot the points correctly and were aware that the resulting graph would be a straight line. Some lines inappropriately passed through the origin even at times 'bending' from a correct line through the candidate's four points.

(ii) There was some misreading of the scale in evidence with candidates using $t = 2.1$, although most candidates identified the vertical intercept within a reasonable range of values. This was the value of ln*P* rather than *P* and some did not make the required step of anti-logging this.

(iii) This part was best solved by using the line drawn and reading off the values. Many candidates successfully did this and stated their answers with little working. Others wrote down a general equation of a line and then substituted in values and this tended to be less accurate. It was not uncommon to see the reciprocal of the gradient being calculated.

(iv),(v) These two parts seemed to work together for most candidates or in many case were omitted together. Both parts required the candidate to use the previous part but, as has been indicated above, the wording needed to be read carefully and in this case this was often not the case. Instead, many chose two points from the original data and substituted into the given relationship and solved the two resulting equations. While this may be condoned where the final values were sufficiently accurate it leads to a loss of credit otherwise. In the final part in was not uncommon to see candidates trying to solve either an equation or an inequality without using the given value of 1000 or ln1000.

Answers: **(ii)** 20.1 **(iii)** $m = 1.28$ $c = 0.2$ **(iv)** $a = 1.22$ $b = 3.60$ **(v)** 5.3

Question 11

(i) Many correct solutions to this part were seen. Better candidates set out the steps clearly, working always on the left side, until the right side appeared only in the last step. Candidates need to be aware that it is not good practice to keep the right side present in every line of working as at least one of the required steps, usually an identity, will be assumed rather than used. This is even more of a problem if the right side is used with the left to rearrange the given identity as if it were an equation. Some solutions relied on 'side' working to quote identities for example. This can make the solution disjointed particularly if this working is interspersed with the intended solution.

(ii) Full marks were often obtained in this part also. A significant number of candidates missed the solutions arising from cos$x$ = –0.5. As the solutions appeared in all four quadrants on this occasion it was not always clear that candidates had considered the two cases separately. This question indicated that the previously proven result should be applied. Some candidates used their partial working instead and arrived at a quadratic in tan$x$ which was usually solved correctly.

Answers: **(ii)** 60 300,120 240

Question 12

(i) There were some very well presented and accurate solutions to this question. Most candidates were able to gain some credit for this part but few had fully correct solutions. The majority realised that the displacement needed differentiating to find an expression for the velocity although the sign and/or coefficient of the derived cosine term were not always accurate. This should then have been equated to zero for instantaneous rest and solved. Many solutions substituted $t = 0$ at this stage resulting in no further progress being possible. Candidates should be aware that these equations are only valid when t is measured in radians and therefore working in degrees was not condoned at this stage. Candidates should also be aware of the context of the question and that time cannot be negative. Inappropriate values of t were quite frequent. On this occasion it was best to evaluate the two relevant values for the displacement separately rather than as limits to an integral. The second instance had a smaller displacement than the first as the particle changed direction and the limits approach gave a negative outcome. Throughout this question candidates could gain marks if they explicitly showed how they had solved or substituted at the various stages. Where values appeared without evidence credit could only be allowed if the values shown were correct.

(ii) When a candidate had correctly differentiated in the first part then they usually correctly differentiated a second time to find an expression for the acceleration. Similarly slips in signs and coefficients were also usually repeated. The substitution was mostly correct. Candidates should be aware that this value of t should give an exact value for the acceleration. Candidates are also advised to check their calculator mode as the use of π degrees was not uncommon.

Answers: **(i)** 0.487 **(ii)** –25

ADDITIONAL MATHEMATICS

Paper 4037/22
Paper 22

Key messages

To succeed in this examination, candidates need to give clear, logical answers to questions and show sufficient method so that marks can be awarded. Candidates need to take note of instructions in questions such as 'Without using a calculator,' or 'You must show all your working'. These instructions indicate that omitting method will result in a significant loss of marks. Candidates who omit to show key method steps in their solution, perhaps because they have used a calculator, risk losing a significant number of marks should they make an error. This is the case whether or not the use of a calculator is permitted for a particular question. Candidates should ensure that their answers are given to no less than the accuracy demanded in a question. When no particular accuracy is required in a question, candidates should ensure that they follow the instructions printed on the front page of the examination paper. Careful attention should be given to the accuracy required for angles in degrees, which varies from those in radians.

General comments

Some candidates seemed to be well prepared for this examination. They showed good understanding of basic concepts and many candidates were able to apply techniques successfully.

Clear and neat solutions often indicate logical thinking. Candidates whose presentation was good often scored more highly. Other candidates need to understand that, when their work is difficult to follow, it is difficult for marks to be awarded. Sometimes candidates misread their own writing when their presentation is poor and accuracy is, unnecessarily, lost. Some candidates used extra sheets of paper to carry out 'rough calculations'. These calculations were usually then deleted and annotated as rough work. This work was often poorly presented and difficult to assign to particular questions. On occasion, it is possible to award marks for rough work that has been deleted and not replaced. This can only be done if the work is readable. Candidates who use rough paper can also miscopy their own work. For these reasons, candidates would do better to write all of their working as part of their main solution to a question.

Many candidates showed all their method clearly, step by step. Other candidates could improve by showing a complete method, rather than a partial method. This is essential if a question asks candidates to 'Show that…'. This instruction indicates that information about the answer has been given. In these cases, the marks are either awarded for the method leading to the answer or for the full and correct mathematical justification of a given statement. The need for this was highlighted in **Questions 5(i)**, **6**, **8(iii)**, **8(v)** and **12(i)** in this examination. Showing clear and full method is also very important when the use of a calculator is not allowed, as was the case in **Questions 2** and **3**.

Some candidates would also do better if they read the question more carefully and focused on key words such as 'Hence, …'. The use of this instruction indicates that the previous part or parts of the question should be used. This is often because a particular method is being assessed. Candidates should be aware that when they choose to ignore this key word they may be penalised.

Candidates seemed to have sufficient time to attempt all questions within their capability.

Comments on specific questions

Question 1

Some candidates were able to produce concise, full and accurate solutions very quickly. The method of squaring both sides of the given equation was very successful for those candidates that used it, as it resulted in a more efficient solution. Most candidates attempted to form and solve linear equations. Some candidates found a correct pair of solutions twice as they wrote and solved four linear equations rather than two. This was acceptable although it did waste some time. Other candidates solved $5x + 3 = 1 - 3x$ and then negated both sides, finding $-\frac{1}{4}$ from both equations and not attempting to solve any others. Candidates who did this needed to realise that they had simply formed another version of the same equation they had already solved. Other candidates would have perhaps improved if they had checked their solutions in the given equation as sign slips were made and extra or incorrect values were stated. Some candidates misunderstood the nature of the absolute value and thought that the answers -2, $-\frac{1}{4}$ needed to be made positive and gave their final answers as 2 and $\frac{1}{4}$.

Answer: $x = -2,\ -\frac{1}{4}$

Question 2

Many candidates were able to demonstrate successfully that they had the necessary manipulation skills to find the correct value. A variety of excellent solutions were seen. Candidates who dealt with the negative nature of the power by inverting the given expression in the first step often were the most successful. Those candidates who left this step until later in the solution sometimes forgot to invert or did so incorrectly. Most candidates showed sufficient working for it to be clear that they had not used a calculator. To be convinced that a calculator has not been used, at least three terms in the numerator of an attempt to rationalise an irrational denominator needed to be shown. Similarly, when squaring, at least three terms were considered acceptable evidence of manual calculation. The solution for this question was multi-step and clear evidence of working without a calculator had to be seen at each stage in the process for full marks to be awarded.

Answer: $9 - 4\sqrt{5}$

Question 3

In this question, candidates were given the opportunity to show that they were able to factorise a cubic expression without using their calculator to find roots. In order to do this, it was expected that candidates would show clear and full method to find a linear factor and then use that linear factor in order to find and factorise a correct quadratic factor. Many candidates earned full marks for their solutions. The first mark was commonly awarded for the correct use of the factor theorem for $x = 2$. Stating f(2) = 0 was not sufficient for the mark to be awarded as the substitution of 2 into the expression needed to be seen. Stating 'by trial and improvement method' without showing any method was insufficient for the mark to be awarded. Some candidates successfully used synthetic division. More candidates used formal long division and their success depended on how careful they were with signs, as a few slips were seen. Very many candidates were able to find a correct quadratic factor. 'Factorise' was a key instruction in this question. Some candidates changed the expression given into a polynomial equation by writing $10x^3 - 21x^2 + 4 = 0$. These candidates almost always went on to solve the equation they had formed. This was penalised and these candidates need to have an improved understanding of the difference in demand between the instructions 'factorise' and 'solve'. It was clear from sight of factors such as $\left(x - \frac{1}{2}\right)$ or $\left(x + \frac{2}{5}\right)$ that some candidates were using their calculators to find roots and then working back to state factors. This approach should be discouraged.

Answer: $(x - 2)(5x + 2)(2x - 1)$

Question 4

A good number of candidates found this to be a very accessible question and earned all 6 marks. The most commonly used successful method was to differentiate the equation of the curve, find the gradient of the tangent using the equation of the normal and then equate and solve. A small number of candidates equated the equation of the tangent in the form $y = 5x + c$ and the curve. They then used the discriminant of the resulting quadratic equation to find c. These few candidates were also often successful. The most common incorrect method seen was to equate the equations of the normal and curve and consider the discriminant of the equation that resulted. These candidates needed a deeper understanding of the relationship between the tangent to a curve, the normal to a curve and the gradient of a curve at a point. A few other candidates found the coordinates of the turning point of the curve, rather than those of P.

Answer: $x = 2$, $y = 9$, $k = 47$

Question 5

(i) Most candidates attempted to apply the product rule. Some candidates did this neatly and accurately, with the components of the product listed and differentiated correctly and the general pattern of the product rule stated. Candidates who lost the accuracy mark often did so as they differentiated $\ln 5x$ as either $\frac{1}{5}$ or $\frac{1}{5x}$. A small number of candidates multiplied out before applying the product rule. This was acceptable, although not recommended, as it introduced an extra step in the method and increased the likelihood of making a sign or arithmetic error. The presentation of some answers was very poor. Many candidates corrected errors and the corrections made were not always clear. It may have helped these candidates if they had written their solutions out again on extra paper, rather than attempting to compress their extra work into the space given. Some candidates stated a correct form of the answer following wrong working. They usually had extra terms that did not cancel and so it was clear that an error had been made. These candidates needed to check back through their method to make the necessary correction. Some candidates needed to take more care with brackets. The accuracy mark was not awarded to candidates who omitted brackets that were necessary for expressions to be unambiguous. Other candidates needed to take more care with signs and decimals, as slips were fairly common.

(ii) This part of the question was intended to lead candidates into **part (iii)**. Very many candidates were able to make a correct statement. Some candidates left their answer as $\ln (5x)^3$, misinterpreting the instruction which was to write the expression in terms of $\ln 5x$. Other candidates gave the answer $9\ln 5x$ after stating $\ln 5^3x^3$.

(iii) This part of the question proved to be a good discriminator. The keyword 'Hence, …' indicated that **part (i)** and **part (ii)** needed to be used to answer this part of the question. Only the most able candidates were successful in making all the connections required. Some candidates were able to show a partial manipulation of the information from **part (i)**, for example $\int \left(x^4 \ln 5x\right) dx = -0.2x^5(0.2 - \ln 5x)$. Other candidates divided the expression by –6 rather than dividing by –2 and multiplying by 3, as required. Commonly, incorrect attempts involved integrating the product term by term or treating expressions in x as if they were constants, bringing them out of the integral as multipliers.

Answers: **(i)** $-2x^4\ln 5x$ **(ii)** $3\ln 5x$ **(iii)** $-\frac{3}{2}\left(0.4x^5(0.2-\ln 5x)\right)+c$

Question 6

Many candidates were able to apply a correct technique to answer this question. The most common method was to state and manipulate the discriminant of the given equation. Whilst many candidates did this correctly only a few were able to show that the required condition was satisfied or successfully state a conclusion. Some candidates did not observe that $p^2 + 2pq + q^2$ could be factorised to $(p + q)^2$. These candidates usually attempted to make an incomplete argument based on p^2 and q^2 both being positive. Other candidates did state $(p + q)^2$ then stated the incorrect condition $(p + q)^2 > 0$, either in words or symbols. A few candidates used the quadratic formula or factorised and found the roots of the equation. Most of these omitted to state any conclusion about the roots they had found. The weakest solutions seen usually involved candidates substituting values for p and q and showing that the roots of the equations they had formed were real. Verification of one or two cases was, and always will be, insufficient to prove a general statement of this type.

Question 7

(a) Both parts of this question were very well answered. Most candidates demonstrated good understanding of the definition of a logarithm and were able to change the exponential form of the equation into the correct logarithmic form.

(b) In this part of the question candidates were required to rewrite a logarithmic equation in exponential form in order to solve it. Again, the majority of candidates were able to access this question. Some candidates increased the complexity of what needed to be done by changing the base. This was often done correctly even though it was not necessary.

(c) A good number of fully correct answers were seen for this part of the question. Some candidates omitted the negative solution. The most common successful approach was to convert all terms to powers of 2 and then collect the powers. A small number of candidates successfully worked with powers of 4 and a few correctly separated the power of 32 and formed the relationship $8^{x^2} = 8^3$. A few candidates made invalid combinations of terms. For example, $32^{x^2-1} = 64^{x^2}$ was a common first step from weaker candidates**.** Some candidates made their first error when collecting the powers. Examples of this were $\dfrac{5x^2 - 5}{2x^2} = 4$ and $5x^2 - 5 = 4 \times 2x^2$. Again, these were commonly seen in weaker answers.

Answers: **(a)(i)** 7 **(ii)** $\dfrac{1}{7}$ **(b)** $y = \dfrac{1}{3}$ **(c)** $x = \pm\sqrt{3}$

Question 8

(i) Very many excellent solutions were given for this part of the question. Almost all candidates earned full marks. A small number of candidates found and used the coordinates of the mid-point. This was not necessary and sometimes resulted in errors being made.

(ii) Again, a very high proportion of candidates were able to use the correct points and a correct application of Pythagoras' theorem to find the required length.

(iii) As this was a 'Show that…' type of question, it was necessary for completely justified statements to be made for full marks to be awarded. Candidates were not allowed to assume the angle *ADC* was a right angle and so, in all coordinate based solutions, gradients or lengths needed to be found independently using coordinates before the proof could be concluded using an appropriate rule or relationship. The most efficient method of solution for this question was to find the coordinates of the point *D*, find the gradient of *CD* and then apply the rule for perpendicular gradients. This was the neatest solution and many candidates used it. In this case, it was important to use or imply the use of the rule for perpendicular gradients. Stating that $-\dfrac{2}{3}$ and $\dfrac{3}{2}$ are perpendicular gradients, alone, was insufficient as a conclusion. It was necessary for candidates to state the relationship between perpendicular gradients, at the very least. Equal numbers of candidates chose to use the longer and more calculation-heavy Pythagoras' theorem approach. Exact lengths were essential for this method to be valid. In order to earn the final mark candidates using the squared values needed to provide integer values and those working with lengths needed to work with surds and show that two sides could be combined to give the third.

A small number of candidates attempted arguments based on congruent triangles. These were seldom successful as many lacked the necessary vigour of this type of proof and facts were stated without necessary justification or were simply assumed. This approach needed a great deal more explanation which, for some candidates, is often more challenging.

(iv) This part of the question was well-answered with many correct vectors stated. A few candidates would have improved by reading the question a little more carefully or by understanding the term 'position vector'. These candidates tended to state $\begin{pmatrix} 4 \\ -7 \end{pmatrix}$ as their answer. Other weak answers were based on $\begin{pmatrix} -8 \\ 8 \end{pmatrix} + \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ -7 \end{pmatrix}$ and consequently $\begin{pmatrix} 12 \\ -15 \end{pmatrix}$ was a common wrong answers from these candidates.

(v) Better candidates carefully considered what was needed for a fully justified argument and provided enough evidence for full marks to be awarded. Many different successful and complete arguments were made. Some candidates gave a partially correct argument. This was often a comparison of a pair of opposite gradients or lengths of sides. At the start of this question, candidates were instructed that 'Solutions to this question by accurate drawing will not be accepted.' Regardless of this, a small number of candidates still made sketches or reasonably accurate diagrams and suggested that this was sufficient as a proof. These diagrams were not acceptable.

Answers: **(i)** $y = -\frac{2}{3}x + \frac{8}{3}$ **(ii)** $\sqrt{208}$ **(iv)** $\begin{pmatrix} -4 \\ 1 \end{pmatrix}$

Question 9

(i) Many candidates were able to complete the square successfully. A high proportion of candidates earned full marks and a good number of candidates were able to earn 2 marks. A fairly common error from those who were not fully correct was to make a bracketing error and so have an incorrect constant. A few candidates did not give an expression of the required format.

(ii) A few excellent graphs were seen. Better candidates took note of the restricted domains of f and f^{-1} and were able to draw the correct portion of each graph. These candidates then indicated the locations of the key points of each graph, or at least one of the graphs, as required. Better candidates chose equal scales on each axis and this made the symmetry of the two graphs in the line $y = x$ much easier to achieve. Other candidates often would have improved if they had related the domain of the function to the graph of the function. Also, some candidates need to understand that any key points on a graph, such as intercepts and turning points, should be indicated where possible. Candidates had been asked to write $f(x)$ in a form from which the turning point could easily be identified; it was expected that this would be marked as a key point on the sketch.

(iii) This question proved discriminatory and many candidates found it challenging. A few excellent and complete solutions were seen. Candidates who had taken note of the restricted domain in **part (ii)** often continued to use that in this part of the question, as was required. Very many candidates earned a mark for swapping the variables at some point and taking a correct first step in rearrangement. Many of these, however, did not consider the negative square root or left their final answer with $\pm$ in front of the square root. A few candidates multiplied out their answer to **part (i)**, not always re-forming the original equation, and attempted unsuccessfully to rearrange that. These candidates clearly had not seen the usefulness of the format of their answer to **part (i)**. Some candidates omitted to give any answer for the domain of the inverse function. The most successful approach in finding the domain of f^{-1} was to consider the range of f. Some candidates did this very well whilst others forgot to change the variable in the statement of the domain from y to x. A few candidates seemed to be attempting to find the domain of the inverse function from their expression for it. This is not advisable as it is dependent on the expression found being correct.

Answers: **(i)** $2(x-1.5)^2+0.5$ **(iii)** $f^{-1}(x) = 1.5 - \sqrt{\frac{x-0.5}{2}}, \quad x \geq \frac{1}{2}$

Question 10

In questions of this type, it is essential that candidates check that their calculator mode is appropriate before answering each part. It is very common for one part of the question to assess working in degrees whilst another part of the question assesses working in radians.

(i) Many candidates scored at least the first 2 marks for successfully rewriting the equation and taking the inverse sine of $\frac{3}{4}$. Only a few candidates made errors with these initial steps either thinking that the sine of a difference was the same as a difference of sines or making premature approximations when finding their reference angle. Candidates who made premature approximation errors needed to understand that they should have given working values to more than 3 significant figures to be sure that their final answers were accurate to 3 significant figures. The rubric on the front page of the question paper indicates that the accuracy required for angles in radians that cannot be stated exactly, is 3 significant figures. Some candidates gave only one angle rather than the two required. Many good candidates avoided this error. They did this by writing down that $0 \leqslant x \leqslant \frac{\pi}{2}$ means that $-\frac{\pi}{4} \leqslant 3x - \frac{\pi}{4} \leqslant \frac{5\pi}{4}$ and/or by drawing a quadrant (CAST) diagram or a simple sine curve. A few candidates worked in degrees, rounded to 1 decimal place and then converted their answers to radians. This resulted in inaccuracies and this approach is not recommended. A few of these candidates gave answers in degrees. Candidates who did this needed to pay more careful attention to the key words in the question as it was clear from the information in the question that radians were required here.

(ii) A good number of candidates answered this question very well. Many understood the need to write the equation in terms of a single trigonometric ratio and correctly solved the resulting quadratic equation. Some candidates made the question far more challenging than was necessary by using more than one trigonometric identity. As the simplest and most efficient identity to use was provided for candidates in the formulae list on page 2, candidates were expected to form a correct form of the equation as a first step. A few candidates correctly replaced $\sec^2 y$ by $1+\tan^2 y$ and then incorrectly replaced $\sec y$ by $1+\tan y$. Some candidates made errors in manipulation when working with their more complicated expressions and other candidates were never able to arrive at an equation in terms of a single trigonometric ratio. Those that did were often able to factorise or solve their equation correctly. Some candidates made substitutions to simplify the factorisation of the quadratic. Many of these candidates stated what substitution they had used and they were often correct. Some candidates did not state the substitution they had used and stated, factorised and solved an equation in *x*, for example. These candidates then sometimes incorrectly went on to state $\sec^2 y = 5$ following $x = 5$. A consequence of this was that the factorisation in *x* was then incorrect and could not be credited. A few candidates stated that sec *x* was $\frac{1}{\tan x}$, while others stated $\frac{1}{\cos} y = 5$ and were unable to make any further progress. Very few candidates showed any evidence of checking their angles in the original equation. A check such as this would have helped candidates who had erroneous angles in their solutions, such as 258.5°, to discard them. A small number of candidates formed a cubic equation in cos *y*, rather than a quadratic. This equation had an extra factor of cos *y* which usually was not discarded. It should have been clear from their working that it was not possible for cos *y* to be 0, so again, a check may have helped here. Premature approximation errors were less frequent in this part.

Answers: **(i)** 0.544, 1.03 radians **(ii)** 78.5°, 281.5°

Question 11

(i) A very high proportion of candidates gave a correct answer to this part. A few candidates did more work than was necessary, writing all the terms over a common denominator. A small number of candidates incorrectly used the coordinates of *B* to state a value for the constant of integration. This was not valid and it seemed to occur when candidates confused '$+ c$' when used as the constant of integration with its use as the *y*-intercept in the general equation of a straight line. Only a very few candidates differentiated or misapplied the general rule for integrating powers of *x*.

(ii) Many correct answers were seen for this part of the question. Often working was incomplete or missing completely. Candidates who did not show working risked losing all the marks if their answers were incorrect. Some candidates stated the final area of the larger rectangle as –25, for example, and this was not acceptable. A few candidates, having found the correct coordinates required for the calculation of the areas, then proceeded to apply Pythagoras' theorem to find the lengths they needed, rather than understanding that the coordinates were sufficient. This sometimes resulted in errors. Weaker solutions involved differentiating the equation of the curve and equating to 5, equating the equation of the curve to 0 and solving or using the formula for the area of a triangle rather than a rectangle. Sign errors were also seen on occasion. Weaker candidates sometimes attempted to integrate in this part, not understanding that this was not possible without limits. A few candidates totalled the areas of the rectangles. These candidates needed to have read the question more carefully.

(iii) A few fully correct and accurate solutions were seen. Candidates needed to apply a correct strategy to combine the areas of the rectangles found in **part (ii)** and the areas under the curve from –5 to 0 and 0 to 1 in this part of the question. Candidates were instructed to show all their working. As is usually the case in these questions, the substitution of the limits needed to be seen and done in the correct order. Some candidates treated –5 as the upper limit rather than the lower limit. Candidates need to be aware that, when 0 is one of the limits, it is not always the lower limit. Many candidates omitted to show the substitution of the limits at all. A good number of candidates were able to earn the mark for the application of a correct strategy. A few good candidates made premature approximation errors and, as a result, their final answer was not accurate. The need to consider the areas of the rectangles and not to simply subtract the equations of the line and curve was indicated by the use of the key word 'Hence, …' in the question. Some candidates needed to give more careful attention to this, as the simplified pair of integrals with 3 terms in each formed from the differences of equations resulted in a more straightforward solution. Candidates using this approach were penalised. The use of the modulus should not have been necessary in this question. Some candidates attempted to use it to convert negative areas found from wrong working into positive areas. Values found from wrong working were not credited. The most common wrong solution offered was to integrate the curve from –5 to 1. These candidates almost always ignored the areas of the rectangles, although on occasion 30 was subtracted. A few candidates used methods that were far more complicated than necessary, adding and subtracting several additional areas. These were sometimes successful even though the likelihood of making an error was far greater.

Answers: **(i)** $\frac{x^4}{4}+\frac{4x^3}{3}-\frac{5x^2}{2}+5x+c$ **(ii)** *OEAB* = 25, *OBCD* = 5 **(iii)** $\frac{886}{12}$

Question 12

(i) A good number of candidates differentiated correctly. A few candidates did not complete their chain rule, forgetting to multiply by 2. Some candidates used the quotient rule. This was more prone to errors as, often, candidates did not differentiate 3 correctly. Many candidates who did differentiate correctly stopped at that point and made no further comment. This meant that their argument was incomplete as they had not fully justified the derivative always being negative. Many candidates who did attempt to comment often stated that the numerator was negative without any reference to the denominator. As the denominator was not constant, candidates needed to make comment about why it was always positive in order to justify the given statement. Some comments were not sufficiently specific for marks to be awarded. A few candidates substituted values into their expression and showed that the result was negative for those few cases. These candidates needed to understand that verification of a handful of values is not sufficient justification for a general statement such as this. Some candidates tried to find the inverse function rather than differentiating. It may be that they did not read the notation carefully enough or it may have been that they misunderstood the notation.

(ii) A reasonable number of candidates stated the correct range. Some candidates used incorrect notation and stated $x > 0$. It is important that the notation used for the range and the domain, when asked for, is correct. Those candidates not giving a correct solution offered various answers with $g > 3$ being fairly common. These candidates many have improved if they had understood how **part (i)** of this question related to this part and if they had thought about the shape of the graph of the function.

(iii) A high proportion of candidates were able to form the correct composite function. Only a few candidates reversed the order of the composition. Some candidates went on to simplify the expression they had found. This was not necessary and sometimes resulted in an error which impacted on **part (iv).**

(iv) Very many excellent solutions were given to this part of the question. Occasionally, candidates equated their expression to 0 rather than 5. Other slips made by those who were not offering fully correct solutions were to evaluate $2(0) + 1$ as 3 rather than 1 or to omit the '$+ 3$'.

(v) This part of the question assessed recall of the knowledge that the domain of a composite function is the domain of the input function of the composition. Here *g* was input into *h* and so the domain of *hg* is the same as the domain of *g*. Those candidates who understood that earned a quick final mark. Some candidates attempted to find the domain from the expression for the composite function only. Some candidates omitted the *x* and made statements such as 'Domain of *hg* is $> -\frac{1}{2}$'. These incomplete statements could not be credited. Other candidates gave their answer as $hg > -\frac{1}{2}$. Again the notation used here was incorrect. Candidates need to be aware of the importance of using the correct notation for domain and range.

Answers: **(i)** $\frac{-6}{(2x+1)^2}$ **(ii)** $g > 0$ **(iii)** $\frac{3k}{2x+1} + 3$ **(iv)** $k = \frac{2}{3}$ **(v)** $x > -\frac{1}{2}$